

Reg. No.: _____

Name: _____

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each question carries 3 marks

1. Show that the series $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ is convergent.
2. Find $\frac{d}{dx} (e^x \operatorname{sech}^{-1} \sqrt{x})$
3. Identify the surfaces $5x^2 - 4y^2 + 20z^2 = 0$
4. Equation of a surface in spherical coordinates is $\rho = \sin \theta \sin \phi$
Find the equation of this surface in rectangular coordinates.
5. Given $f = e^x \sin y$; show that the function satisfies the Laplace equation $f_{xx} + f_{yy} = 0$
6. Let $w = 4x^2 + 4y^2 + z^2$, where $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$ Find $\frac{\partial w}{\partial \rho}$
using chain rule.
7. A particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4 \cos \pi t)\mathbf{i} + (4 \sin \pi t)\mathbf{j} + t\mathbf{k}$ Find the displacement of the particle during the interval $1 \leq t \leq 5$.
8. Find the tangent to the curve $r(t) = (t^2 - 1)\mathbf{i} + t\mathbf{j}$ at $t = 1$
9. Evaluate $\int_1^a \int_1^b \frac{dy dx}{xy}$
10. The line $y = 2 - x$ and the parabola $y = x^2$ intersect at the points $(-2, 4)$ and $(1, 1)$. If R is the region enclosed by $y = 2 - x$ and $y = x^2$, then find $\iint_R (y) dA$

(10 x 3 = 30 Marks)

PART B

Answer any 2 complete questions each having 7 marks

11. Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.
12. Test the convergence of $\frac{x}{12} + \frac{x^2}{23} + \frac{x^3}{34} + \dots$
13. Find the Taylor's series of $\frac{1}{x}$ about $x = 1$.



Answer any 2 complete questions each having 7 marks

14. Find the domains of (i) $f(x, y) = \sqrt{25 - x^2 - y^2 - z^2}$ (ii) $f(x, y) = \ln(x - y^2)$ and describe them in words.
15. Find the limit of $f(x, y) = \frac{-xy}{x^2 + y^2}$ as $(x, y) \rightarrow (0, 0)$ along (i) the X-axis, (ii) the Y-axis (iii) the line $y = x$.
16. Find the spherical and cylindrical coordinates of the point that has rectangular coordinates $(x, y, z) = (4, -4, 4\sqrt{6})$

Answer any 2 complete questions each having 7 marks

17. Locate all relative maxima, relative minima and saddle point if any, of $f(x, y) = y^2 + xy + 4y + 2x + 3$
18. Let f be a differentiable function of 3 variables and suppose that $W = f(x - y, y - z, z - x)$. Prove that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.
19. Find the local linear approximation $L(x, y)$ to $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ at the point $P(4, 3)$. Compare the error in approximating 'f' by L at the specified point $Q(3.92, 3.01)$ with the distance between P and Q.

Answer any 2 complete questions each having 7 marks

20. Find $y(t)$ where $y''(t) = 12t^2i - 2tj$, $y(0) = 2i - 4j$, $y'(0) = 0$.
21. Find the arc length parametrization of the line $x = 1 + t, y = 3 - 2t, z = 4 + 2t$ that has the same direction as the given line and has reference point $(1, 3, 4)$.
22. Find the directional derivative of $f(x, y) = e^x \sec y$ at $P(0, \pi/4)$ in the direction of PQ where Q is the origin.

Answer any 2 complete questions each having 7 marks

23. Find the area bounded by the x-axis, $y = 2x$ and $x + y = 1$ using double integration.
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.
25. Sketch the region of integration and evaluate the integral $\int_1^2 \int_y^{y^2} dx dy$ by changing the order of integration.